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Technical Notes

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Stabilization of a Mach 5.92 Boundary Layer by Two-Dimensional Finite-Height Roughness

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I. Introduction

THE performance of hypersonic transportation vehicles and reentry vehicles is significantly affected by the laminar-turbulent transition of boundary-layer flows over vehicle surfaces as transition has a first-order impact on lift, drag, stability, control, and surface heating of these vehicles. For example, the roughness-induced transition is an important consideration in the design of thermal protection systems [1,2]. For a reentry vehicle transition can lead to an increase in surface-heating rate by a factor of five or more. Hence, the understanding of transition mechanisms is critical to the development of future hypersonic vehicles [3].

The dominant mechanism in the roughness-induced transition is the transient growth associated with purely wall-normal and spanwise velocity perturbations, which finally evolve into streak-like motion. Transient growth has been studied by many researchers. Andersson et al. [4] calculated the transient growth of a flat-plate boundary layer to steady disturbances and found that the maximum transient growth scales linearly with the distance from the leading edge, which was consistent with the results of Hanifi et al. [5]. Collis and Lele [6] investigated the stationary crossflow vortices in a three-dimensional boundary layer over a swept wing due to surface roughness near the leading edge. The results showed that the initial amplitude of crossflow vortices is enhanced by convex surface curvature and strongly reduced by nonparallel flow effects. White and Ergin [7] studied the transient growth of a Blasius boundary layer generated by a spanwise array of roughness elements. The results indicated that the energy of the roughness-induced disturbances is proportional to the roughness-height-based Reynolds number. White et al. [8] further investigated the effects of the height and diameter of cylindrical roughness element on transient growth. Their results showed that the energy of transient growth is proportional to the square of the roughness-height-based Reynolds number. In addition, transient growth strongly depends on roughness diameter. Choudhari and Fischer [9,10] examined the transient growth in a laminar boundary layer due to a spanwise array of circular disks at the surface. The effects of roughness height, size, and shape were explored. Their results indicated that the energy of transient growth is consistent with the scale of White et al. [8].

One explanation to the roughness-induced transition is the transient growth theory pioneered by Reshotko and Tumin [11] where the roughness-induced velocity perturbations are assumed to be proportional to roughness height. Wang and Zhong [12] studied the transient growth of a flat-plate boundary layer to small surface roughness periodic in the spanwise direction. The results showed that the assumption, the roughness-induced velocity perturbations are proportional to roughness height, is only valid when roughness height is smaller than approximately one thirty-fifth of the local boundary-layer thickness. Therefore, the transient growth theory cannot be directly applied to the transient growth induced by finite-height roughness [8–10].

Unfortunately, the roughness-induced transition in hypersonic boundary layers, especially that induced by arbitrary surface roughness, is still poorly understood due to the limitations in experimental facilities and numerical methods [13]. Marxen et al. [14] studied the effect of a two-dimensional (2-D) roughness element on the stability of a hypersonic boundary layer. They showed that in the vicinity of the separation regions located near the edges of the roughness the destabilization of a second-mode disturbance occurs for a certain frequency. Even though roughness is generally expected to lead to an early transition several past studies have shown that roughness can stabilize boundary-layer flows and delay transition in some situations. For example, Holloway and Sterrett [15] concluded from their experiments that surface roughness can delay transition when its height is less than the local boundary-layer thickness. Fujii [16] found that a wavy roughness can suppress the transition of a Mach 7.1 boundary layer. Although no physical explanation is given the repeatable results indicate that some unknown mechanisms must have been presented. Recently, Wang et al. [17] studied the response of a Mach 8 flow over a 5.3 deg half-angle wedge to wall blowing–suction and showed that the synchronization point plays a critical role in the stability of hypersonic boundary layers. The results of Wang et al. [17] indicate that the location of roughness with respect to the synchronization point might play a role in the roughness-caused second-mode destabilization [14] and transition delay [15,16].

In this Technical Note, we present our numerical simulation results on the stabilization of a Mach 5.92 flow over a flat plate by 2-D finite-height roughness. The main objective is to investigate the effect of roughness location on mode S instability. Although three-dimensional surface roughness is more relevant to the roughness-induced transition the current numerical simulations on 2-D finite-height roughness are still valuable for the effect of roughness on the stability of hypersonic boundary layers. Figure 1 shows a schematic of the problem where boundary-layer waves are excited by a blowing-suction slot near the leading edge and a finite-height roughness is put downstream of the slot. Specifically, four cases of numerical simulations are conducted with the roughness at different locations. The results show that mode S is destabilized only when the roughness element is located closely upstream of the synchronization point. When the roughness element is downstream of the synchronization

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point mode S is stabilized. The current study provides a possible explanation for the roughness-induced transition delay and suggests a possible way to delay transition by putting roughness elements downstream of the synchronization point.

II. Governing Equations and Numerical Method

For the current numerical simulations the governing equations are the 2-D Navier–Stokes equations in the following conservation-law form,

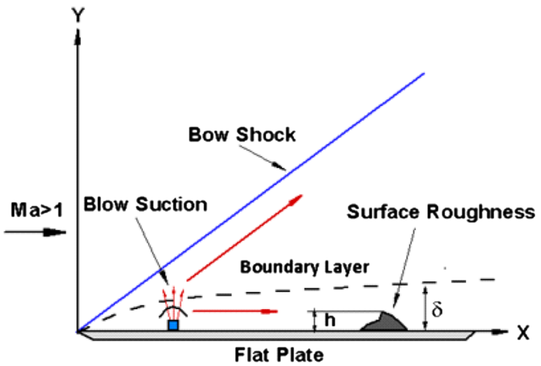


Fig. 1 A hypersonic flow over a flat plate with blowing-suction and roughness.

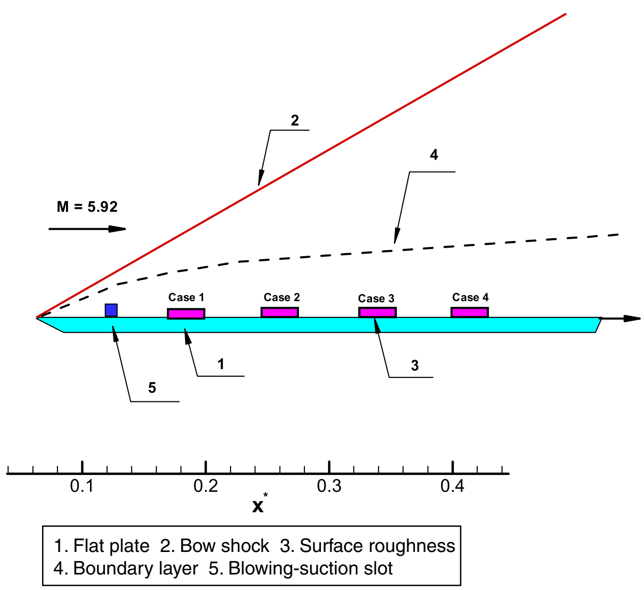


Fig. 2 A schematic of different roughness locations for the four cases of simulations.

$$\frac{\partial U}{\partial t} + \frac{\partial F_j}{\partial x_j} + \frac{\partial F_{vj}}{\partial x_j} = 0 \tag{1}$$

where U, F_j and F_{vj} are the vectors of conservative variables, convective and viscous fluxes in the jth spatial direction, respectively, i.e.,

$$U = \{\rho, \rho u_1, \rho u_2, e\} \tag{2}$$

$$F_j = \begin{Bmatrix} \rho u_j \\ \rho u_1 u_j + P\delta_{1j} \\ \rho u_2 u_j + P\delta_{2j} \\ (e+p)u_j \end{Bmatrix} \tag{3}$$

$$F_{vj} = \begin{Bmatrix} 0 \\ \tau_{1j} \\ \tau_{2j} \\ \tau_{jk}u_k - q_j \end{Bmatrix} \tag{4}$$

In our simulation only perfect-gas flow is considered, i.e.,

$$P = \rho R T \tag{5}$$

$$e = \rho\left(c_v T + \frac{1}{2}u_k u_k\right) \tag{6}$$

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\lambda\frac{\partial u_k}{\partial x_k} \tag{7}$$

$$q_j = -k\frac{\partial T}{\partial x_j} \tag{8}$$

where R is the gas constant. The specific heat C_v is a constant determined by a given ratio of specific heats γ. The viscosity coefficient μ is calculated by Sutherland's law,

$$\mu = \mu_r\left(\frac{T}{T_0}\right)^{3/2}\frac{T_0 + T_s}{T + T_s} \tag{9}$$

where $\mu_r = 1.7894 \times 10^{-5}$ Ns/m^2, $T_0 = 288.0$ K, $T_s = 110.33$ K, and λ is assumed to be $-2\mu/3$. The heat conductivity coefficient k is determined by a constant Prandtl number.

The new high-order cut-cell method of Duan et al. [18] is used for numerical simulations. Due to the difficulty in grid generation around arbitrary surface roughness as shown in Fig. 1, it is advantageous to use a fixed-grid cut-cell method. The new method combines a non-uniform-grid finite-difference scheme for irregular grid points near surface roughness and a hock-fitting scheme for the bow shock.

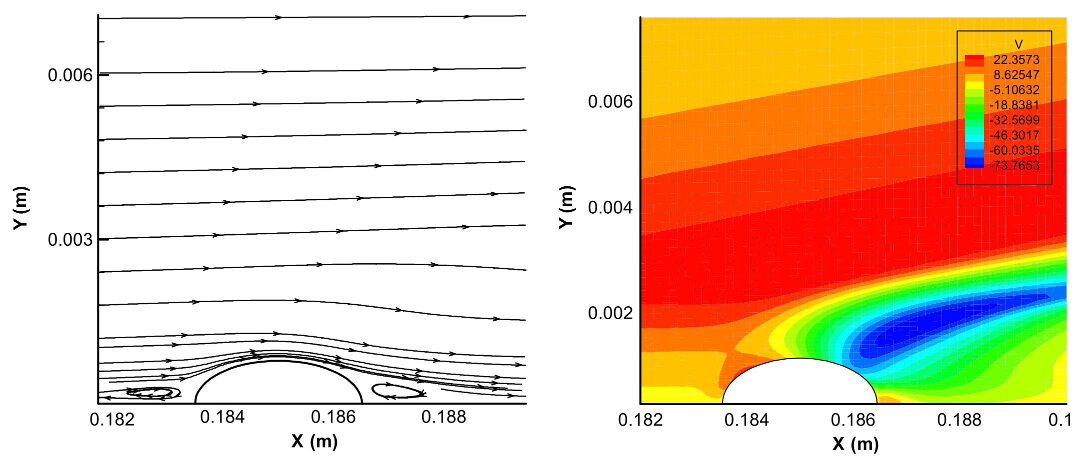


Fig. 3 Streamline pattern and wall-normal velocity contour around the roughness element.

Details of the numerical method are omitted and can be found in Duan et al. [18].

III. Flow Conditions and Roughness Model

The freestream flow conditions are the same as those used in Wang and Zhong's numerical simulation [12,19], i.e.,

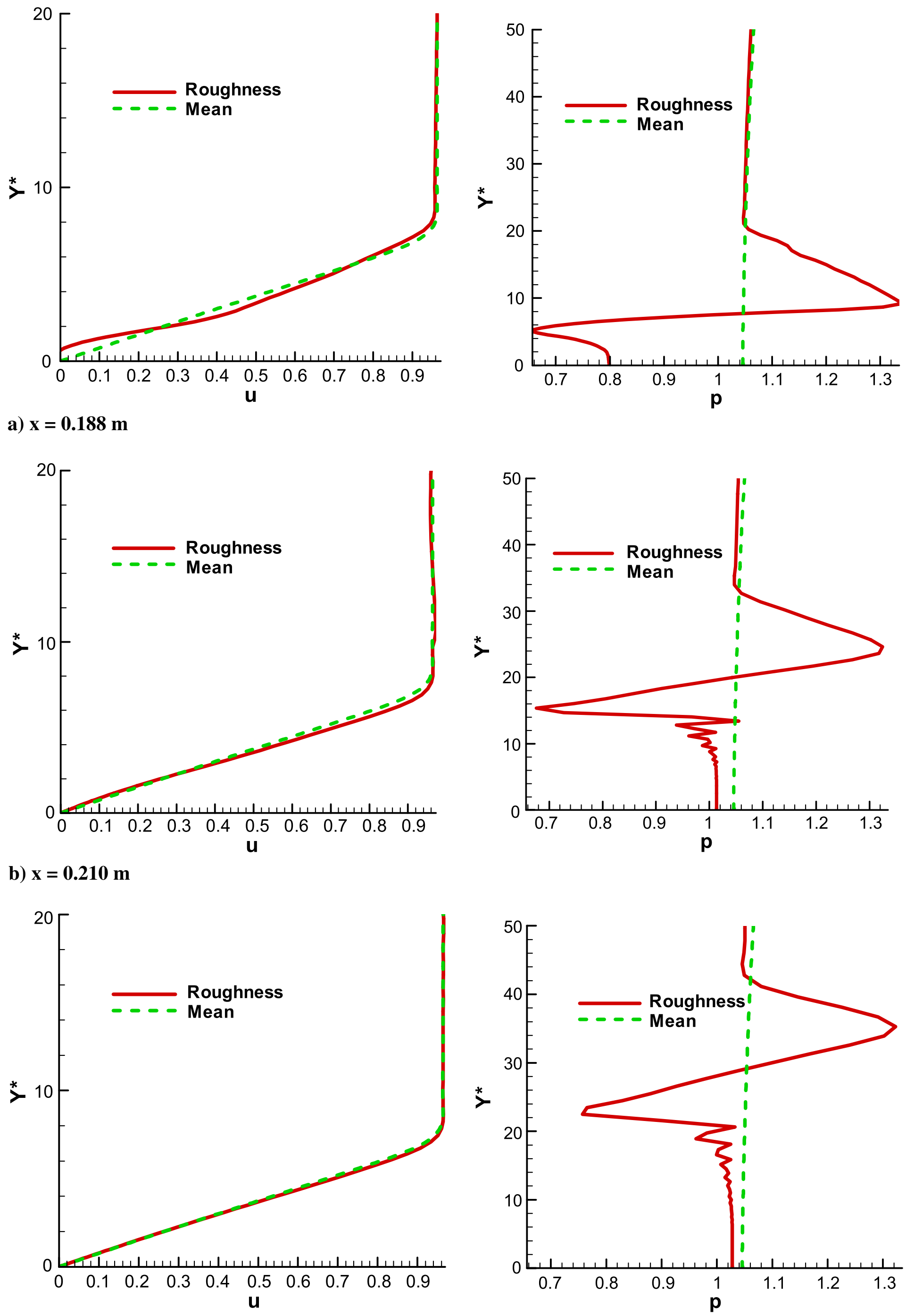
$$\begin{cases} \mathrm{Pr}=0.72, \quad R_\infty=\rho_\infty u_\infty/\mu_\infty=13.0\times10^6/\mathrm{m}, \quad \gamma=1.4 \\ M_\infty=5.92, \quad T_\infty=48.69\ \mathrm{K}, \quad P_\infty=742.76\ \mathrm{Pa}, \quad f=100\ \mathrm{kHz} \end{cases} \tag{10}$$

where M_∞, T_∞, P_∞, Pr, R_∞ are Mach number, temperature, pressure, Prandtl number and unit Reynolds number, respectively. In the current simulation, the flat plate has a length of 1.69 m and is assumed to be adiabatic. Unless stated otherwise flow variables are presented as dimensionless. We nondimensionalize flow velocities by freestream velocity u_∞, density by ρ_∞, pressure by $\rho_\infty u_\infty^2$, and temperature by T_∞.

Similar to Whitehead's experiments [20] the surface roughness is chosen to be a 2-D bump. The shape of roughness is defined by the following elliptic equation,

$$\frac{(x-x_c)^2}{a^2}+\frac{y^2}{b^2}=h^2 \tag{11}$$

where $a=2$, $b=1$. The location and height of surface roughness is determined by x_c and h.



a) x = 0.188 m

b) x = 0.210 m

c) x = 0.240 m

Fig. 4 Distributions of streamwise velocity and pressure in the wall-normal direction at three locations.

IV. Effect of Roughness Location on Boundary-Layer Stability

Numerical results of the steady base flow and the stability characteristics for the boundary layer over a smooth surface have been reported in previous papers [12,19]. The pressure perturbation along the flat plate for the boundary layer without roughness has been obtained in a previous study where the perturbation was induced by wall blowing-suction [19]. In this section the effect of the roughness on the steady base flow and the stability characteristics is discussed. Four cases of numerical simulations have been carried out for the roughness located upstream or downstream of the synchronization point.

Figure 2 shows a schematic of different roughness locations. Specifically, the location and height of roughness element are as follows:

Case 1: $x_c = 0.185$ m, $h = \frac{1}{2}\delta = 0.00081$ m

Case 2: $x_c = 0.260$ m, $h = \frac{1}{2}\delta = 0.00111$ m

Case 3: $x_c = 0.331$ m, $h = \frac{1}{2}\delta = 0.00141$ m

Case 4: $x_c = 0.410$ m, $h = \frac{1}{2}\delta = 0.00171$ m

For wall blowing-suction at the frequency of 100 kHz the synchronization point is located at $x = 0.33184$ m. Therefore, the roughness element is upstream of the synchronization point in cases 1 and 2, near the synchronization point in case 3, and downstream of the synchronization point in case 4.

Figure 3 shows the steady base flow around the roughness element obtained from the case 1 simulation. The streamline pattern and wall-normal velocity contour show the features of the steady base flow due to the roughness. Because the flow is supersonic behind the bow shock a family of Mach waves is generated over the roughness. The compression waves near the upstream edge of the roughness are followed by expansion waves when the flow expands near the downstream edge of the roughness. These Mach waves are approximately parallel to the bow shock when they are propagating further downstream.

Figure 4 shows the distributions of streamwise velocity and pressure in the wall-normal direction for case 1 simulation at three different locations downstream of the roughness. Compared with the mean flow over a smooth surface the steady base flow is distorted significantly right after the roughness. However, the modification of the steady base flow decreases as the flow moves downstream. Further downstream after $x = 0.240$ m the Blasius boundary-layer solution is reestablished. The feature of pressure distortion is moving upward as the flow moves downstream because Mach waves are propagating away from the flat plate. The result indicates that surface roughness only has local effects on the steady base flow.

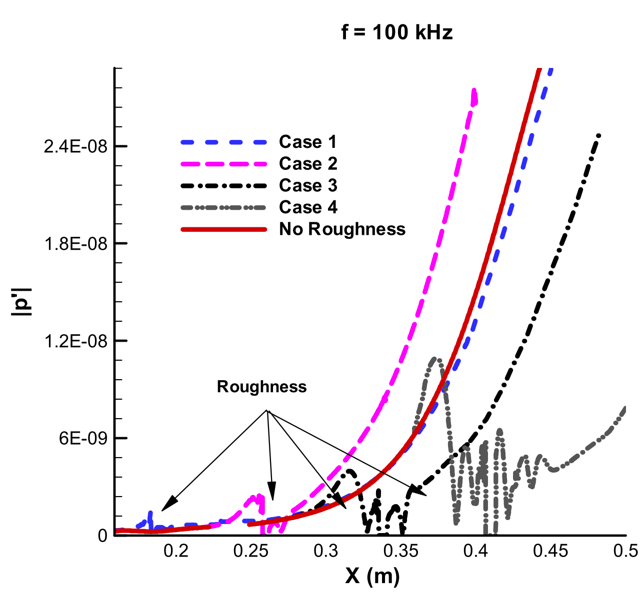
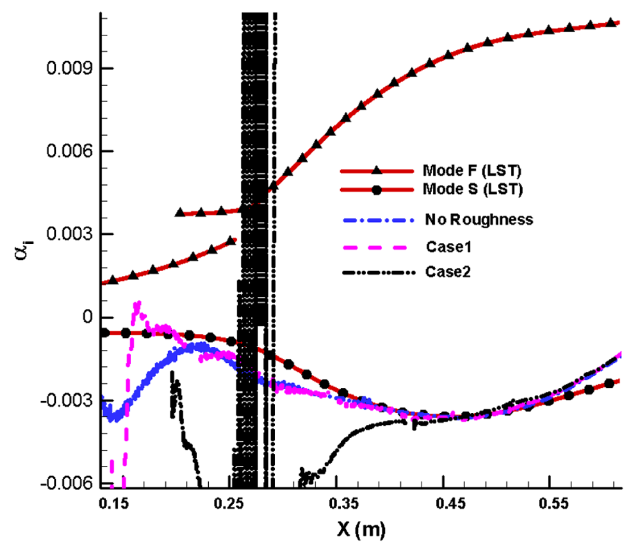


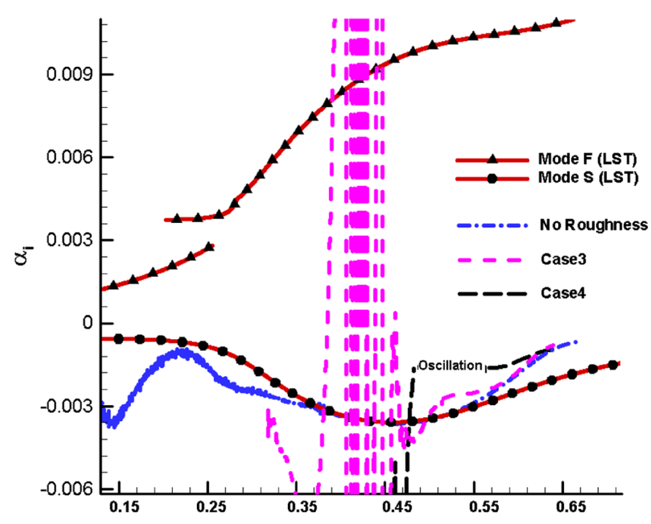
Fig. 5 Comparison of the pressure perturbation amplitude along the flat plate.

Figure 5 compares the pressure perturbation amplitude along the flat plate obtained from the four cases of numerical simulations. It shows that the development of pressure perturbation far downstream of the roughness element is essentially the same as that in the boundary layer without roughness. Previous results have demonstrated that the amplified perturbation is dominated by unstable mode S [19]. It is noticed in Fig. 5 that mode S is first disturbed around the roughness element and then restarts its exponential growth when it propagates further downstream. The result indicates that surface roughness only has local effects on the instability of mode S. When the roughness is located far upstream of the synchronization point (case 1), mode S develops in a similar way as in the boundary layer without roughness. As the roughness moves downstream (case 2) the amplitude of pressure perturbation along the flat plate is amplified, i.e., mode S is destabilized when the roughness is in the close upstream region of the synchronization point. In cases 3 and 4 the roughness is mainly downstream of the synchronization point. The roughness is found to stabilize mode S. For the last two cases the pressure perturbation is still amplified further downstream, but the strength is significantly weaker than that in the first two cases.

Figure 6 compares the imaginary part of wave number α_i obtained from numerical simulations with that predicted by the linear stability theory (LST) based on the boundary layer without roughness. Here



a) For simulations of cases 1 and 2



b) For simulations of cases 3 and 4

Fig. 6 Comparison of α_i obtained from numerical simulations with that predicted by LST.

the numerical α_i is calculated from the pressure-perturbation amplitude along the flat plate. It is noticed that the numerical result does not agree well initially with that of LST due to the coexistence of various continuous waves and discrete modes in the boundary layer right after the blowing-suction slot. After $x > 0.33$ m mode S in the boundary layer is fully developed and becomes dominant. Therefore, the numerical α_i agrees well with the theoretical prediction of Mode S. In the region around the roughness element the pressure perturbation along the flat plate oscillates strongly for all four cases, which causes the oscillations of the numerical α_i. But further downstream of the roughness the steady base flow is restored and the numerical α_i of Mode S is smooth.

For case 1, the numerical α_i does not change much by the roughness. This shows the roughness only has local effects on the instability of Mode S. But for case 2 mode S is destabilized around the roughness or in the region before the synchronization point. Thus, the modified steady base flow amplifies the amplitude of pressure perturbation. Downstream of the roughness element mode S develops in a similar way as in the boundary layer without roughness. For case 3 and 4 where the roughness is mainly located after the synchronization point mode S is stabilized as the pressure-perturbation amplitude decreases. The stabilization effect of the finite-height roughness on mode S is also shown in Fig. 6, where the growth rate $(-\alpha_i)$ for both cases is significantly lower than that predicted by LST in the region after the surface roughness. The growth rate for all four cases with surface roughness agrees well with that predicted by LST further downstream. This again validates that the roughness element only affects the local instability of Mode S.

Overall, the numerical results show that roughness location plays an important role on the development of mode S excited by the blowing-suction slot. Mode S is destabilized only when the roughness is in the close upstream region of the synchronization point. When the roughness element is downstream of the synchronization point mode S is stabilized. The development of pressure perturbations downstream of surface roughness is essentially the same as that in the boundary layer without roughness because surface roughness only has local effects on the instability of mode S.

V. Conclusions

In this technical note we present numerical results on the stability of a Mach 5.92 flow over a flat plate to wall blowing-suction with the effect of 2-D finite-height roughness. Four cases of numerical simulations are conducted with the roughness locating upstream or downstream of the synchronization point. The results indicate that roughness location plays an important role in the developments of mode S excited by the blowing-suction slot. Mode S is destabilized only when the roughness element is located in the close upstream region of the synchronization point. On the other hand, when the roughness element is downstream of the synchronization point mode S is stabilized. In addition, surface roughness only has local effects on the steady base flow and the stability characteristics. The relationship between the roughness location and the synchronization point suggests that a possible way to delay boundary-layer transition by putting the roughness element downstream of the synchronization point is feasible. The current study also provides a possible explanation for the roughness-induced transition delay. To understand the flow physics behind the current findings further systematic studies are needed.

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